Reduction of Computation Load for Digital Beamforming of a Cluster of Beams Using the Complex Conjugate Property of Beam Weights

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Abstract:
This paper presents a way to reduce the computation load in Digital Beamformer(DBF) for active phased array radars having multiple simultaneous beams which are organized as clusters. The beam cluster has weights which are complex conjugate pairs. The proposed method utilizes this property to reduce the number of multiplications needed for implementation by half. This however requires the same number of additions or subtractions in the computation as in the conventional way.

Key words: Digital Beamformer, Cluster of Beams

I. Introduction

In the conventional way, in a digital beamformer(DBF) each beam is formed individually by multiplication of complex weight(for that beam for the selected element) with each element output and summation of all these. The beam formation algorithm is summarized as follows:

Output Y= CX

$$Y=\begin{bmatrix} y1 \\ y2 \\ y3 \\ .. \\ .. \\ yM \end{bmatrix}$$

$$Y=\begin{bmatrix} c1,1 & c2,1 & .. & cN,1 \\ c1,2 & c2,2 & .. & cN,2 \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ c1,M & c2,M & .. & cN,M \end{bmatrix} \mathrm{x} \begin{bmatrix} x1 \\ x2 \\ x3 \\ .. \\ .. \\ xN \end{bmatrix}$$

Where
X= Input at any instant of time which is a complex matrix with entries corresponding to each element output.
C= Coefficient matrix for the selected frequency for M beams
N= total number of elements (separate channels) in the system and
M= total number of beams

Hence to form M beams we have to do, N*M complex multiplications in the conventional way.

II. New Method

The beam cluster has complex weights which are complex conjugate pairs. This is derived below.
For element with location m,n for the beam with offsets(beam_offset_u , beam_offset_v) in (u,v) space, the Beam weights are computed as

$$\text{Weight} = e^{-j*2*pi/\lambda*(m*dx*beam\_offset\_u+ n*dy*beam\_offset\_v)} \quad (1)$$

Where

λ	the Wavelength
m, n	the positional coordinate of the element
dx, dy	the inter-element spacing in x and y.

With substitution, $k=2*pi/\lambda$ and
$k_1=m*dx$, $k_2= n* dy$, "(1)" reduces to

$$\text{Weight}= e^{-j*k*(k_1*beam\_offset\_u+ k_2*beam\_offset\_v)} \quad (2)$$

With substitution,
$k_3=k*(k_1*beam\_offset\_u+ k_2*beam\_offset\_v)$, "(2)" reduces to

$$\text{Weight} = e^{-j*k_3} \quad (3)$$

For the same element, for the complementary beam with beam-offset(-beam_offset_u , -beam_offset_v) in (u,v) space, the beam weights are

$$\text{Weights}= e^{-j*2*pi/\lambda*(m*dx*-beam\_offset\_u+ n*dy*-beam\_offset\_v)} \quad (4)$$

With substitution, $k=2*pi/\lambda$ and
$k_1=m*dx$ and $k_2= n*dy$, "(4)" reduces to

$$\text{Weight}= e^{-j*k*(-k_1*beam\_offset\_u+ -k_2*beam\_offset\_v)} \quad (5)$$

With substitution,
k_3=k*(k_1*beam_offset_u+ k_2*beam_offset_v), "(5)"
reduces to ,

$$\text{Weight} = e^{+j*k_3} \qquad (6)$$

From (3) and (6), the weights are complex conjugates for the selected element for beams with offsets (beam_offset_u , beam_offset_v) and(- beam_offset_u , -beam_offset_v).

III. EXAMPLES

1 EXAMPLE-1

Suppose we have to form 12 beams as a 6 x2 cluster. Assume that (0,0) point is in the center as shown in Fig.1. So the beam offsets are [(-du,5dv), (-du,3dv), (-du, dv), (-du,-dv), (-du,-3dv), (-du, -5dv), (du,5dv), (du,3dv), (du, dv), (du,-dv), (du,-3dv), (du, -5dv)] for B1 to B12 respectively. So B1=*B12, B2=*B11, B3=*B10, B4=*B9,B5=*B8 and B6=*B7.

2 EXAMPLE-2

Suppose we have to form 9 beams as 3 x 3 cluster. Beam offsets are [(0 ,0),(2du ,0),(-2du ,0),(-2du, 2dv),(0, 2dv),(2du,2dv),(-2du,-2dv)(0, -2dv), (2du ,-2dv)] .Therefore, B4=*B9, B3=*B2,B5=*B8 B7=*B6. B1 does not need any multiplication (offsets are zero); we just have to add all the element contributions.

3 EXAMPLE-3

Suppose we have to form 6 beams as 2 x 3 cluster. Assume that (0,0) point is in the center as shown in Fig.3. Beam offsets are [(-2du, dv), (0,dv), (2du, dv), (-2du, -dv), (0,-dv), (2du, -dv),for B1 to B6 respectively. So B1=*B6, B2=*B5 and B3=*B4.

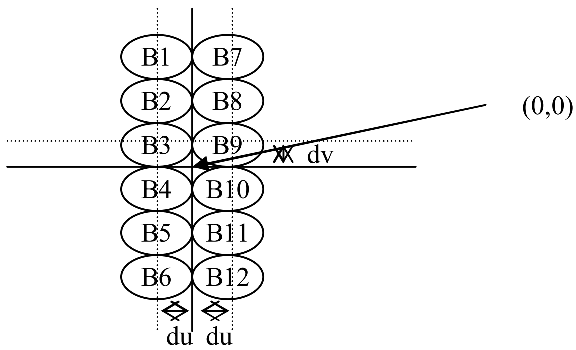

Figure 1. Beam offsets for 6x 2 cluster beams

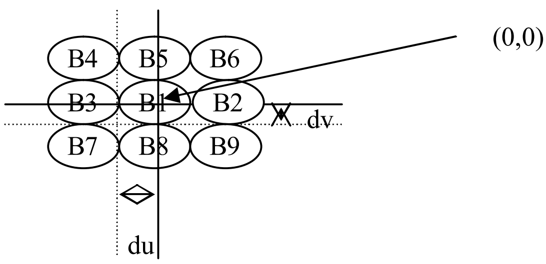

Figure 2. Beam offsets for 3 x 3 cluster of beams.

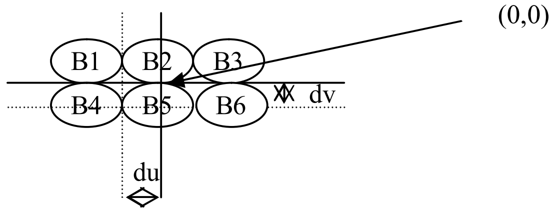

Figure 3. Beam offsets for 2 x 3 cluster of beams.

IV. IMPLEMENTATION

The beam cluster has complex weights which are complex conjugate pairs as derived earlier. Assume radar with multiple-beams organized as a cluster shown below:

$$\begin{bmatrix} b11 & b12 & .. & b1k \\ b21 & b22 & .. & b2k \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ bj1 & bj2 & .. & bjk \end{bmatrix}$$

That is there are ' j' rows and 'k' columns of beams.

In the above cluster ,
b_{11} coefficient(weights)= complex conjugate of b_{jk},
b_{12} coefficient(weights)= complex conjugate of $b_{j(k-1)}$
....
b_{21} coefficient(weights)= complex conjugate of $b_{(j-1)k}$
....
b_{j1} coefficient(weights)= complex conjugate of b_{1k}

The beam weights are existing as complex conjugate pairs and hence instead of multiplying each element with a weight for each beam we can share the multiplier for the conjugate beams.
Suppose we have to form 12 beams as a 6 x2 cluster as shown in Fig.1. So B1=*B12, B2=*B11 etc.

Hence in the implementation, we can use the same multiplier output, but change operation to subtraction instead of addition. This saves the number of multipliers to half.
Suppose element 1 output is

$$E1= a+jb \qquad (7)$$

where 'a' and 'b' are the ' I' and 'Q' components of E1.
For beam1(B1) let the weight(W_B1) be

$$W\_B1= c+jd \qquad (8)$$

Then for beam B12 the weight(W_B12) will be c-jd as B1= *B12;

$$W\_B12= c-jd \qquad (9)$$

From (7) and (8), E1 contribution to B1 can be computed as

$$(a+jb)(c+jd)=(ac-bd)+j(ad+bc) \qquad (10)$$

Similarly from (7) and (9), E1 contribution to B12 can be computed as

$$(a+jb)(c-jd)=(ac+bd)+j(-ad+bc) \qquad (11)$$

Form (10) and (11) we can notice that we need only the same real multipliers 'ac', 'bd', 'ad' and 'bc' for E1 contribution to B1 and B12 . Only the sign in the addition operation changes. That is instead of sum we have to form the difference.

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V. CONCLUSION

In general, the computation load is only half- the number of multipliers needed for the conventional beamforming method. However the same number of subtraction/addition operations is needed. For every radar, depending on the beam cluster, we have to identify the conjugate coefficients and use it to reduce the number of multiplications.

BIO DATA OF AUTHOR

Sindhu.P received the B.Tech degree in Electronics and Communication Engg. from University of Calicut in 1999.

She is working in Bharat electronics Ltd since 1999. She has a total of about fourteen and a half year D&E experience in BEL. She was working in the Sonar System Division on Sonar Transmission signal generators, Signal Noise Simulator and Receivers. Since Feb 2012 she is working in Central D&E- Radar Signal Processing group on Generic DBF.